# **An Improved Adaptive Multi-Objective Particle Swarm Optimization for Disassembly Line Balancing Problem**

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## **I. Introduction**

DLBP is a multi-objective problem that was first described by Gungor and Gupta [1] and has been mathematically proven to be NP-complete by McGovern and Gupta [2]. Exhaustive search works well enough in obtaining optimal solutions for small-sized instance. However, its exponential time complexity limits its application to large sized instances [3]. Although some researchers have formulated the DLBP using mathematical programming techniques [4-5], they quickly become unsolvable for a practical-sized problem due to their combinatorial nature [3]. According to the above reasons, heuristic approaches to attain a (near) optimal condition with respect to objective functions are widely used to solve DLBP.

## **The DLBP model description**

## **Problem definition and formulation**

This paper deals with the most extensively studied disassembly line balancing problem known as a deterministic and single-model of product that undergoes complete disassembly. Based on concept and assumptions made by Gungor and Gupta [6], the disassembly line balancing problem can be defined that a discarded product P is composed of  $n$  parts and a part removal is considered as a task  $OP_i$ . The tasks sequence is then utilized to group the tasks into workstations of which the number is  $m$ . The precedence relationships considered are of *AND* type and are represented using the immediately preceding matrix  $\left[ X_{ij} \right]_{n \times n}$  where:

1 0 *i j Xij* , i f t ask OP i mmedi at el y pr eccdes t ask OP , ot her wi se

In order to state the partition of total tasks, we use the assignment matrix 
$$
\left[ Y_{jk} \right]_{n \times m}
$$
, where:  

$$
Y_{jk} = \begin{cases} 1, & \text{if } m\text{th } t \text{ ask}(\mathbf{O}^j) \text{ is assigned to the } k\text{th } st \text{ at } i \text{ on} \\ 0, & \text{of } h \text{er } w \text{ se} \end{cases}
$$

In this paper, the multi-objective DLBP mathematical programming model seeks to fulfill four objectives:

A major constraint is to minimize the number of workstations for a given cycle time *CT* (the maximum time available at each workstation), this objective is represented as:

$$
\min f_1 = m \tag{1}
$$

The balance measure is represented as:

 $\sum_{i=1}^{m} [CT - t_i]^{2}$  $\min f_2 = \sum^m$  $f_2 = \sum_{i=1}^{m} [CT - t_i]^2$ (2)

The third objective is given as follows:

$$
\min f_3 = \sum_{i=1}^n (i \times h_{PS_i}), \ h_{PS_i} = \begin{cases} 1 & \text{hazar dous} \\ 0 & \text{of her w see} \end{cases} \tag{3}
$$

A demand measure was developed to removal of high demand parts early in the part removal sequence. This measure is represented as:

$$
\min f_4 = \sum_{i=1}^n (i \times d_{PS_i}), \ d_{PS_i} \in N, \forall PS_i \tag{4}
$$

Subject to:

$$
\sum_{k=1}^{m} Y_{jk} = 1, j = 1, 2, \cdots n
$$
 (5)

$$
\left[\frac{\sum_{i=1}^{n} t_i}{CT}\right] \le m^* \le n \tag{6}
$$

$$
Y_{ik} \le \sum_{k=1}^{m} Y_{jk}, \forall (i, j) \in IP
$$
\n
$$
(7)
$$

Equation (5) indicates that each task must be assigned to only one station. Constraint (6) guarantees that the number of stations with a workload does not exceed the permitted number. Constraint (7) imposes the restriction that all the disassembly precedence relationships between tasks should be satisfied.

#### **II. IAMOPSO approach for the DLBP**

#### **Arithmetic coding and decoding**

According to the permutation of *n* numbers, a priority based zero in-degree topological sorting technique [7] is used to decode the permutation to FDS. This algorithm selects the task from the candidate tasks based on priority and zero in-degree. In the procedure, a task is an available task only if it has not already been assigned to a workstation and all of its predecessors have already been assigned to a workstation.

#### **Defined evolutionary states**

*Ensured the evolution factor parameter*

The evolution factor parameter can be introduced to reflect the diversity and evolutionary status of the population, as detailed in the following equations.

The mean distance of each particle  $i$  to all the other particles is as follows.

$$
d_i = \frac{1}{NUM - 1} \sum_{j=1, j \neq i}^{NUM} \sqrt{\sum_{k=1}^{D} (x_i^k - x_j^k)^2}
$$
 (8)

Using the  $p_u$  represents the evolution factor parameter by the following equation:

$$
p_u = \frac{d_g - d_{\min}}{d_{\max} - d_{\min}} \in [0, 1]
$$
\n
$$
(9)
$$

Where, the  $d_{\text{min}}$  and  $d_{\text{max}}$  are minimum and maximum mean distance, respectively.  $d_g$  is the mean distance of globally best particle to all the other particles.

## *Fuzzy classification*

This paper partitions the  $p_{\mu}$  into four sets, including exploration, exploitation, convergence, and jumping out in each generation.

Exploration: the set is denoted by  $S_1$  and the fuzzy membership function is defined as follows in detail.

$$
\mu S_1(p_u) = \begin{cases} 0, & 0 \le p_u \le 0.4 \\ 5 * p_u - 2, & 0.4 \prec p_u \le 0.6 \\ 1, & 0.6 \prec p_u \le 0.7 \\ -10 * p_u + 8, & 0.7 \prec p_u \le 0.8 \\ 0, & 0.8 \prec p_u \le 1 \end{cases}
$$
(10)

Exploitation: the set is denoted by  $S_2$  and the fuzzy membership function is defined as follows in detail.

$$
\mu S_2(p_u) = \begin{cases} 0, & 0 \le p_u \le 0.2 \\ 10 * p_u - 2, & 0.2 \prec p_u \le 0.3 \\ 1, & 0.3 \prec p_u \le 0.4 \\ -5 * p_u + 3, & 0.4 \prec p_u \le 0.6 \\ 0, & 0.6 \prec p_u \le 1 \end{cases}
$$
(11)

Convergence: the set is denoted by  $S_3$  and the fuzzy membership function is defined as follows in detail.

$$
\mu S_3(p_u) = \begin{cases} 1, & 0 \le p_u \le 0.1 \\ -5 * p_u + 1.5, & 0.1 \le p_u \le 0.3 \\ 0, & 0.3 \le p_u \le 1 \end{cases}
$$
(12)

Jumping-out: the set is denoted by  $S_4$  and the fuzzy membership function is defined as follows in detail.

$$
\mu S_4(p_u) = \begin{cases} 1, & 0 \le p_u \le 0.7 \\ 5 * p_u - 3.5, & 0.7 \prec p_u \le 0.9 \\ 1, & 0.9 \prec p_u \le 1 \end{cases}
$$
(13)

*Adjust adaptively the control of PSO parameter*

Adjust adaptively the control of PSO parameter is given in Table1.

<b>Table 1:</b> The assembly the control of 1 bo parameter			
<b>Evolution</b> state	$c_{1}$ $(c_{\text{max}} = 2.5, c_{\text{min}} = 1.5, c_1 = 2)$	$(c_{\text{max}} = 2.5, c_{\text{min}} = 1.5, c_1 = 2)$	w
Exploration	$c_1 = c_1 + a, (a \in [0.05, 0.1])$	$c_2 = c_2 - a$ , $(a \in [0.05, 0.1])$	
Exploitation	$c_1 = c_1 + 0.5*a, (a \in [0.05, 0.1])$	$c_2 = c_2 - 0.5 * a$ , $(a \in [0.05, 0.1])$	
Convergence	$c_1 = c_1 + 0.5*a, (a \in [0.05, 0.1])$	$c_2 = c_1 + 0.5*a, (a \in [0.05, 0.1])$	$w = \frac{1}{1+1.5 * e^{-2.6 * p_u}}$
Jumping-out	$c_1 = c_1 - a$ , $(a \in [0.05, 0.1])$	$c_2 = c_2 + a$ , $(a \in [0.05, 0.1])$	

**Table 1.** Adjust adaptively the control of PSO parameter

## **Information entropy**

Suppose that the number of FDS is  $NUM$ , in which includes element of D numbers,  $R_j$  represents a set including the  $j^h$  element of all FDSs,  $b_{ij}$  is a number of repeated values in the  $R_j$ .  $P_{ij}$  represents the occupancy of the  $j^h$  element of  $i^h$  FOS in the  $R_j$ . The information entropy of the  $j^h$  element in the all FDSs can be calculated by following formula:

$$
H_{i} = \sum_{j=1}^{D} \frac{P_{ij}}{b_{ij}} \log_2 \frac{1}{P_{ij}}
$$
 (14)

The diversity of the FDSs can be expressed by the average information entropy of all FDSs.

$$
H = \frac{1}{NUM} \sum_{i=1}^{NUM} \sum_{j=1}^{D} \frac{P_{ij}}{b_{ij}} \log_2 \frac{1}{P_{ij}}
$$
(15)

## *Selection particle of good diversity*

In order to further enhance the convergence speed and precision of the algorithm,  $\sigma_i$  was introduced to assess the diversity of corresponding particle whether has good diversity by following formula and the smaller the  $\sigma_i$  is the more diversities the corresponding particle has. The particle with good diversity is denoted by  $pbest_{bin}$ .

$$
\sigma_i = H - H_i \tag{16}
$$

## **Proposed MOPSO Algorithm Procedure for DLBP**

The aforementioned procedure is iteratively followed until the maximum number of cycles  $(T_{\text{max}})$ , which is adopted as the termination criteria, has been traced. The program flowchart of the procedure is shown in Figure1.



**Figure 1.** The procedure of improved MOPSO for DLBP

## **III. Numerical results**

The number of the population *NUM* is set to be 100. The size of the external non-dominated *NP* is set to 50, a maximum number of iteration  $T_{\text{max}}$  is set equal to 300, other parameters: *w* is set to 0.729,  $p_c$  is set to 0.1,  $p_m$  is set to 0.4, while  $p_{u1}$  is 0.4 and  $p_{u2}$  is 0.05. We use a practical example to confirm the improved algorithm's effectiveness for DLBP. The example consists of 25 subassemblies and working cycle of the workstation is 600 seconds (CT=600). The precedence relationship and knowledge database are given in Figure2 and Table 2.



Table 2. Knowledge base for 52-part DLBP instance



Two Pareto optimal solutions obtained by DLBP IAMOPSO on different runs are given in Figure3 and Figure4, the function values are as follows:  $f_1 = 7$ ,  $f_2 = 8454.675$ ,  $f_3 = 13966$ ,  $f_4 = 136$  and  $f_1 = 7$ ,  $f_2 = 11130.72$ ,  $f_3 = 14686$ ,  $f_4 = 126$ . The results of computer simulation indicate that the DLBP IAMOPSO has performance and application prospect involving multiple objectives that are optimized simultaneously.



## **IV. Conclusion**

The main intent of this paper is to build a particle dimension of the particle swarm of the total number of tasks, and through particle swarm optimization method obtains the best disassembly sequence. Both theoretical proof and numerical experiments indicate that this algorithm is superior to the other algorithms in terms of the objective values.

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